Essential Mathematics for Neuroscience
Lecture 4 - The Usage of Derivatives

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Finding Maxima, Minima, and Saddle-Points

Let \( f \) be a \((n+1)\) times differentiable function. Then \( f \) has a

- minimum at \( x_0 \) if and only if \( f^{(k)}(x_0) = 0 \) for \( k = 1, \ldots, n \) and \( f^{(n+1)}(x_0) > 0 \) with \((n+1)\) even.

- maximum at \( x_0 \) if and only if \( f^{(k)}(x_0) = 0 \) for \( k = 1, \ldots, n \) and \( f^{(n+1)}(x_0) < 0 \) with \((n+1)\) even.

- saddle-point at \( x_0 \) if and only if \( f^{(k)}(x_0) = 0 \) for \( k = 1, \ldots, n \) and \( f^{(n+1)}(x_0) \neq 0 \) with \((n+1)\) odd.
Estimating the rate of a Poisson distribution

Matlab code of observing k spikes in a time window

- » t = [0:0.001:10];
- » p = 0.05;
- » ind = find( rand(size(t)) <= p);
- » s = zeros(size(t));
- » s(ind) = 1;
- » plot(t,s,’k’)

\[ p(k) = \frac{\lambda^k e^{-\lambda}}{k!} \]
Estimating the rate of a Poisson distribution

- `t = [0:0.001:1];`
- `p = 0.05;`
- `m = 5000;`
- `U = rand(m,length(t));`
- `S = zeros(size(U));`
- `S(U <= p) = 1;`
- `figure`
- `imagesc(1-S); colormap gray;`
- `xlabel('time bins'); ylabel('spike trains')`
- `figure, C = sum(S,2);`
- `K = [1:max(C)];`
- `hist(C,K);`
- `hold on`
- `lambda = p*length(t);`
- `plot(K, m*poisspdf(K,lambda),’r’,’LineWidth’,2); hold off`
Estimating the rate of a Poisson distribution
Estimating the rate of a Poisson distribution
Linear approximation

Approximation of \( \sin(x) \) at \( x_0 = 0 \)
Taylor/MacLaurin Theorem

Theorem

For a differentiable function \( f \) and a \( x \) near \( x_0 \), \( f(x) \) can be approximated by

\[
f(x) \approx f(x_0) + (x - x_0)f'(x_0) + \frac{1}{2}(x - x_0)^2f''(x_0) + \frac{1}{6}(x - x_0)^3f'''(x_0) + \ldots
\]

or in general

\[
f(x) \approx f^{(n)}(x) = \sum_{k=0}^{n} \frac{1}{k!}(x - x_0)^kf^{(k)}(x_0),
\]

where \( f^{(k)}(x_0) \) denotes the \( k \)-th order derivative of \( f \), and \( f^{(n)}(x) \) is referred to as the Taylor series approximation to \( f(x) \) at \( x_0 \).
Plotting the Taylor approximation

- » \( t = [-1:0.01:2]; \)
- » \( x_0 = 1; \)
- » `plot(t,exp(t),'k-','LineWidth',3), hold on`
- » \( f_k = \exp(x_0); \)
- » \( n = 1; \)
- » `for k = 1:10`
- » `plot(t,fk,'-r','LineWidth',3); pause`
- » `plot(t,fk,'-b','LineWidth',3);`
- » \( n = n*k; \)
- » \( f_k = f_k + \frac{1}{n} \frac{(t - x_0)}{k} \cdot \exp(x_0); \)
- » `end`
- » `hold off`