Problem Sheet 6: Integrals

You have one week to solve the following exercises. The solved problem sheets are due 21/Nov/11 at the beginning of the lecture. You may use matlab for plotting. Whenever you use matlab, please also hand in your clearly commented source code.

1. Basic integrals: Solve the following integrals.

(a) \[ A = \int_0^2 x^4 - 2.5x \, dx \]

(b) \[ B = \int_1^2 \sqrt{x} + \frac{1}{2} \, dx \]

(c) \[ C = \int_{-\pi}^{\pi} \sin(4x - \pi) \, dx \]

(d) \[ D = \int_0^{\pi} f(x) \, dx \text{ where } f(x) = \begin{cases} 2x^2 & x \leq 1 \\ 2 & 1 < x \leq 5 \\ 0 & 5 < x \end{cases} \]

(e) \[ E(x) = \int_0^x \lambda e^{-\lambda s} \, ds, \text{ for } \lambda > 0. \text{ What happens as } x \to \infty? \text{ What does that mean for the spiking behavior in a Poisson process neuron (comment in one or two sentences)?} \]

(f) \[ (*) \quad F(x) = \int_{-\pi}^{\pi} \sin^2(nx) \, dx, \text{ for } n \text{ any integer.} \]

(g) For each of the functions above, check your answer by numerically by evaluating the integral in matlab.

2. Integrate and fire neuron: In integrate-and-fire neurons, the speed at which the membrane potential \( V(t) \) returns to its resting potential \( V_\infty \) is given by \( V(t) = V_\infty \left( 1 - \exp \left( -\frac{t}{\tau} \right) \right) \). We suppose \( V_\infty = -70mV \).

(a) Show that the \( V(t) \) satisfies the differential equation \( \frac{dV(t)}{dt} = \frac{1}{\tau} (V_\infty - V(t)), \quad V(0) = 0mV. \)

(b) Calculate the total current flow across the membrane up to time \( t \), \[ A(t) = \frac{1}{\tau} \int_0^t (V(s) - V_\infty) \, ds, \text{ where } R = 10M\Omega. \text{ What is } A(T) \text{ for } T \to \infty? \]

**Hint:** Think about how you can express \( (V(s) - V_\infty) \) in terms of \( \frac{dV(s)}{ds} \)

Using \( V(t) = \int_0^t \frac{dV(s)}{ds} \, ds \) you can solve this task without integration!

(c) Calculate \( A(t) \) if \( V(t) \) is given by \( V(t) = V_\infty \left( 1 + \frac{1}{1+e^t} \right). \)

**Hint:** \( f(t) = \frac{1}{1+e^t} \) for \( f(t) = \tan^{-1} t. \)

Feedback

Please fill out the feedback part and hand it in with your solutions. This will help us to adjust the contents of one lecture and further problem sheets.

Time it took me to complete the problem sheet: ______ min

The last lecture was too slow ☐ ☐ ☐ ☐ too fast ☐ ☐ ☐ ☐

The tasks were easy ☐ ☐ ☐ ☐ difficult ☐ ☐ ☐ ☐

Further feedback: